

## Fall 2019 Math 245 Exam 2

Please read the following directions:

Please write legibly, with plenty of white space. Please fill out the box above as legibly as possible. Please fit your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. The use of notes, calculators, or other materials on this exam is strictly prohibited. This exam will begin at 1:00 and will end at 1:50; pace yourself accordingly. Please remain quiet to ensure a good test environment for others. Good luck!



Problem 4. Let  $x \in \mathbb{R}$ . Prove that  $\lfloor x \rfloor$  is unique; that is, prove that there is at most one  $n \in \mathbb{Z}$  with  $n \leq x < n + 1$ .

Problem 5. Prove that, for every  $n \in \mathbb{N}$ ,  $\sum_{i=1}^n \frac{1}{i(i+1)} = \frac{n}{n+1}$ .

Problem 6. Solve the recurrence with  $a_0 = 2$ ,  $a_1 = 5$ , and relation  $a_n = 2a_{n-1} - a_{n-2}$  ( $n \geq 2$ ).

Problem 7. Suppose that an algorithm has runtime specified by recurrence relation  $T_n = 5T_{n/2} + n^2$ . Determine what, if anything, the Master Theorem tells us.

Problem 8. Prove or disprove:  $\forall n \in \mathbb{Z}, \exists m \in \mathbb{N}, n = m(4 - m)$ .

Problem 9. Prove that  $n^2 - n = \Theta(n^2)$ .

Problem 10. Prove that  $\sqrt{5}$  is irrational.